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Wave-free motions of isolated bodies and the existence of motion-trapped modes

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A motion trapping structure can be defined as a freely floating structure under natural or externally applied restoring forces, on or below the free surface of a heavy fluid extending to infinity in at least one direction, which generates a persistent local time-harmonic oscillation of the fluid of finite energy at a particular frequency, due to its own motion at that frequency. Such an oscillation is termed a motion-trapped mode. In this paper it is shown, using accurate numerical computations, that a submerged circular cylinder making forced time-harmonic two-dimensional heave or sway motions of small amplitude in a fluid of either finite or infinite depth, can create a local flow field in which no waves radiate to infinity at particular frequencies and depths of submergence of the cylinder. By tethering such a buoyant cylinder to the bottom of a fluid of finite depth, using a vertical inelastic mooring line, it is shown, by suitable choice of buoyancy and length of tether, how the cylinder, moving freely under its mooring forces, can operate as a motion trapping structure. Such a cylinder would, if displaced from its equilibrium position and released, ultimately oscillate indefinitely at the trapped mode frequency. This simple geometry is the first example of a submerged isolated motion trapping structure free to move under its natural mooring forces.

1. Introduction

When a rigid body makes small simple harmonic oscillations in a single mode of either translation or rotation in a fluid of infinite extent, it experiences an opposing force or moment exactly out of phase with its acceleration. Its motion can therefore be regarded as taking place *in vacuo* provided the mass or inertia of the body is augmented by an added mass or added inertia which takes account of the presence of the fluid. If the body undergoes small oscillations in a heavy fluid which is bounded by a horizontal free surface which extends to infinity the situation is more complicated. Now, the body, whether partially or totally immersed in the fluid, creates a disturbance on the free surface in the form of waves of the same frequency as the body motion, which in general radiate away from the body to infinity. The force on the body due to the fluid is no longer exactly out of phase with the acceleration of the body but has a component exactly out of phase with the linear or angular velocity of the body which is responsible for the work done in generating the radiated waves. Thus in order to incorporate the effect of the fluid into the equation of motion of the body as before, in addition to including an added mass or added inertia term, a radiation damping term proportional to the linear or angular velocity of the body is also needed.

These frequency-dependent hydrodynamic coefficients are widely used in ship hydrodynamics and sophisticated numerical methods have been developed for computing them. Various properties of these coefficients can be proved, including the result that the damping coefficient is non-negative. That it should be positive is not surprising since this simply reflects the fact that waves generated by the body motion travel away from the body and energy is transported to infinity. What is more surprising is that the damping coefficient can vanish at a particular frequency so that no net work is done over a cycle and no waves radiate away, although there is still a local wave field. Examples of this exist in both two and three dimensions. For example Kyozuka & Yoshida (1981) used axisymmetric wave potentials which did not radiate to infinity to construct axisymmetric partially immersed bodies which were 'wave-free' when making small forced vertical (heave) oscillations. The shape of the bodies was narrower near the free surface than below so that the waves created by the deeper part of the bodies was capable of cancelling those made by the narrower part near the free surface. In two dimensions the simplest shape which exhibits zero damping at a given wavenumber is a thin vertical strip rolling about a point which varies with the wavenumber but always lies above the mid-point of the strip. Again this is to enable the waves created by the longer part of the strip below the point of rotation to cancel the waves created by the shorter part above the point of rotation and closer to the surface. This was illustrated in finite depth by Evans & Porter (1996) and also follows from the early explicit results of Ursell (1948) in infinite fluid depth. In fact, for any partially or totally submerged two-dimensional cylinder which is symmetric about a vertical line, there exists a point of rotation on the line, either within or external to the cylinder, for which the wave radiation vanishes at a particular wavenumber. This follows from the Newman (1976) relations which show that the waves radiated to either infinity by such a cylinder have the same phase in both sway and roll. It follows that a linear combination of the two motions involving two real coefficients can be found such that the far-field wave amplitude vanishes. But such a combination is simply equivalent to a single roll motion about a point on the line of symmetry.

In the next section we shall present new results for a single totally submerged two-dimensional cylinder which exhibits zero damping in either forced heave or sway motions. In § 3 we introduce the idea of a motion trapping structure and show how the submerged cylinder described in § 2 can be moored in a natural way to provide the first example of a single-body motion-trapping structure free to move under natural restoring forces.

2. Vanishing of the radiation damping for a submerged cylinder

An example of a submerged body which exhibits zero damping is a two-dimensional submerged circular cylinder moving in either heave or sway. In infinitely deep fluid the scattering and radiation problems for this geometry display many remarkable features. Thus Dean (1948) showed that the reflection coefficient arising when a plane wave is incident on the fixed cylinder vanishes for all frequencies, cylinder radii, and depth of submergence, a result made rigorous by Ursell (1950). Subsequently Ogilvie (1963) in a long detailed paper considered the radiation problem also and showed *inter alia* that the far-field wave amplitude, added mass and radiation damping coefficients at each frequency were identical in heave and sway. As a consequence he showed that a cylinder constrained to make small circular motions about its centre generated waves in one direction only on the free surface, a phenomenon which subsequently formed the basis for a wave energy absorbing device (Evans *et al.* 1979). He also showed that the added mass could become negative for small submergences, a phenomenon which arises whenever the potential energy exceeds the kinetic energy of the fluid (Falnes & McIver 1985).

One phenomenon which was overlooked by Ogilvie was the vanishing of the radiation damping in heave or sway at certain frequencies also at small values of the

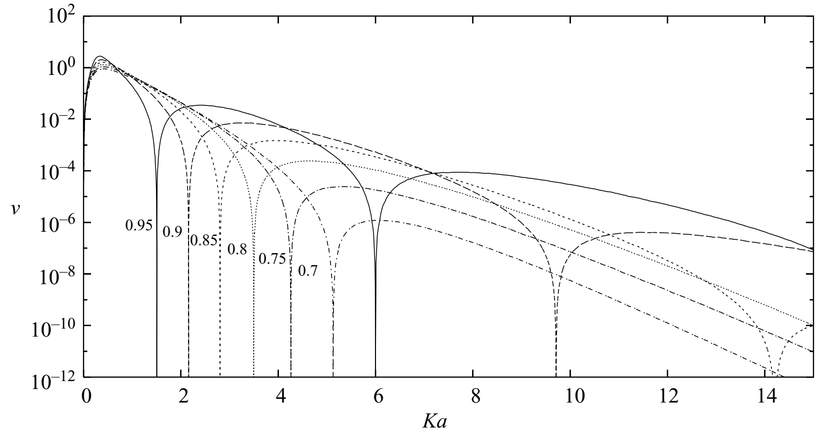

FIGURE 1. Dimensionless radiation damping ν against $Ka = \omega^2 a/g$ for a cylinder, radius a, whose centre is submerged to a depth f, making small-amplitude heave or sway motions. Different curves correspond to $a/f = 0.95, 0.9, 0.85, 0.8, 0.75, 0.7$.

submergence. A close scrutiny of Ogilvie's curves suggests that this appears to be the case but he is reluctant to draw that conclusion, stating only that "Of course this coefficient (ν, the radiation damping) is always positive although its value is very small for certain small values of the depth (of submergence)". We have recomputed the results of the radiation problem and confirmed to a high degree of accuracy that the radiation damping does indeed vanish at certain frequencies. In figure 1, we show a plot of the dimensionless radiation damping coefficient ν against the dimensionless frequency parameter $Ka \equiv \omega^2 a/g$ (ω being the angular frequency and g gravitational acceleration) for different values of a/f between 0.95 and 0.7 where a is the cylinder radius and f is the depth of submergence of its centre. Because the fluid depth is infinite, the curves apply to both heave and sway oscillations, and are calculated using the systems of equations described in the Appendix, with $\nu \equiv \nu^j$, $j = h, s$. The use of a logarithmic scale on the vertical axis shows the zeros of ν by vertical spikes approaching the Ka-axis. These cusp-like features in the curves at zeros of ν contrast with the curves of ν on the log scale approaching the axis at an angle, suggesting an exponential decay in ν. For example, computations show that when $a/f = 0.95$, ν vanishes for $Ka = 1.50394$ and $Ka = 5.9988$ and when $a/f = 0.9$, the corresponding values are $Ka = 2.16016$ and $Ka = 9.7115$.

In figure 2, the added mass for heave and sway in infinite depth is shown, over a much smaller range of Ka. These curves exhibit less interesting behaviour, although it can be seen how the added mass can become negative for cylinders close to the free surface.

The numerical results in figure 1, indicating the presence of zeros of ν, are independently supported by the following robust numerical evidence. At those frequencies for which $\nu = 0$, there are no waves radiating to infinity and so the boundary-value problem for the velocity potential, ϕ, which is defined to satisfy a real kinematic boundary condition on the cylinder surface, is identical to that for $\overline{\phi}$, its complex conjugate. In this case, without loss of generality, we may consider ϕ to be real and in the construction of the solution for ϕ we need only use the real part of the cylinder multipoles provided we ensure finally that there are indeed no waves radiated to infinity. It follows from (A 2) that the real expansion coefficients, b_m, say, will satisfy

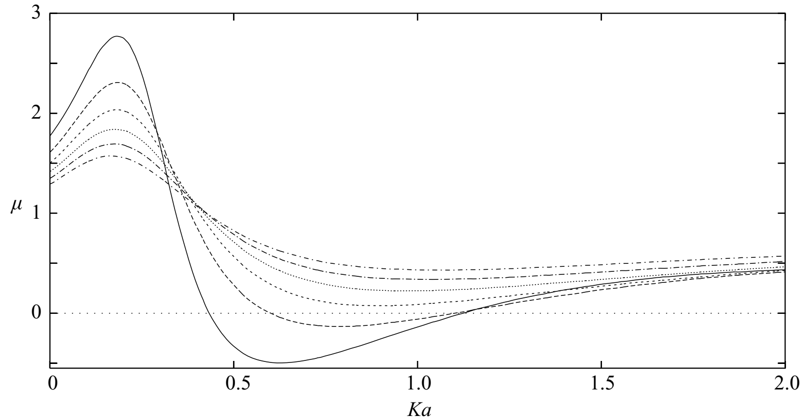

FIGURE 2. Dimensionless added mass μ against $Ka=\omega^2 a/g$, for the same arrangement as in figure 1. Values of a/f range from 0.95 to 0.7 with line styles corresponding to those in figure 1.

the following real infinite system of equations:

$$-\frac{b_m}{m}+\sum_{n=1}^{\infty} b_n \operatorname{Re}\{A_{mn}\} = -\delta_{m1}, \qquad m=1,2,\ldots, \tag{2.1}$$

where A_{mn} refers to either heave or sway in either finite or infinite depth fluid. The wave field at infinity now arises from the pole of the integrand in the real Principal Value integral in the definition of A_{mn} and is a standing wave with amplitude proportional to the real quantity

$$S=\sum_{m=1}^{\infty}\frac{(-Ka)^m}{m!}b_m$$

where the b_m satisfy (2.1) and K needs to be replaced by k, the real positive root of $K=k\tanh kh$, in finite fluid depth, h. Thus, a zero of radiation damping corresponds to $S=0$, and plotting S as a function of the frequency parameter, Ka, for fixed values of a/f generates curves which cross the Ka-axis at exactly those values predicting $\nu=0$ in figure 1. Furthermore, the crossing of the Ka-axis by the curve of S against Ka is robust to changes in the accuracy of the numerical scheme. Note from (A 2) that when $S=0$, and $\nu=0$, the coefficients a_m are clearly real. The technique described above was first used by the authors in a related problem involving the determination of sloshing trapped modes above the cutoff near a fixed vertical cylinder on the centreline of a narrow wave tank (see Evans & Porter 1998).

The method described above is used to plot, in figure 3, the location of zeros of ν in $(Ka, a/f)$-parameter space in infinite depth fluid. It can be seen that the number of zeros increases as a/f tends towards unity. We have been unable to extend the range of accurate results beyond those shown in figure 3 owing to numerical difficulties. For example, for large Ka and small a/f, 'background' values of ν are extremely small and locating zeros requires some quite subtle and high-precision computing, whilst for a/f very close to unity, large truncation sizes are required and matrix inversion becomes ill-conditioned. Although the range of results is limited, it is tempting to conclude from figure 3 that a zero of damping exists for all values of a/f and that the number of zeros increases indefinitely as the cylinder approaches the free surface.

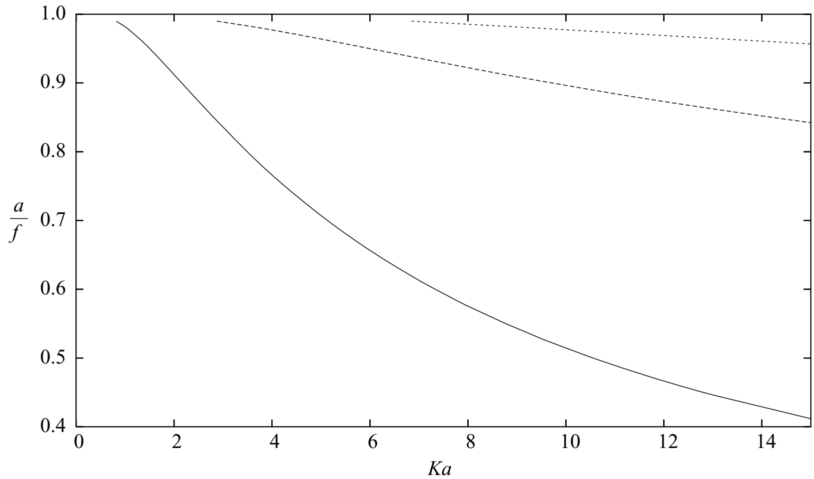

FIGURE 3. Location of zeros of ν in $(Ka, a/f)$-parameter space.

Our numerical results indicate that zeros persist for a/f as small as 0.3, although these zeros are difficult to resolve from background values of ν of the order of 10^{-40}. Indeed, figure 3 is reminiscent of the dispersion curves (McIver & Evans 1985) for edge waves along the top of a long submerged horizontal circular cylinder, where it was shown numerically that the number of edge waves increased indefinitely with decreasing submergence, whilst as the submergence is increased the number of edge waves reduced to just one.

We have also repeated the calculation in finite fluid depth, h, using the extension of the multipole method described in, for example, Linton & McIver (2001) and we have found zeros of damping for each depth (for a cylinder sufficiently close to the free surface). In this case the sway and heave damping and added mass coefficients are no longer the same but the curves are qualitatively similar to figures 1 and 2, with the effect of finite depth being small even for $a/h = 0.4$ (a cylinder occupying 80 % of the depth). The technique described above for the case of infinite depth fluid was used to provide robust numerical confirmation of the zeros of ν.

3. Motion trapping structures

A sloshing trapping structure is a fixed structure which supports a local oscillation or trapped mode at a particular frequency in an unbounded fluid with a free surface. A variety of trapped modes exist in both two and three dimensions. For a review, see Evans & Kuznetsov (1997). In contrast, a motion trapping structure supports a trapped mode whilst oscillating freely at that frequency. In the absence of viscosity such a body would oscillate at that frequency indefinitely with no wave energy being radiated away to infinity. McIver & McIver (2006) have derived the conditions which need to be satisfied for a motion trapping structure. They are that both the radiation damping and any linear damping due to external mooring forces should be zero and also that there should be a balance between the inertia forces on the body, including those due to the surrounding fluid, and any linearized restoring forces, such as hydrostatic forces and any linear external mooring forces. Thus in both two or three dimensions the condition is

$$(M + a(\omega))\omega^2 = \lambda \tag{3.1}$$

where $\omega/2\pi$ is the wave frequency for which zero radiation damping occurs, M is the mass or inertia of the body, $a(\omega)$ its added mass or added inertia, and $\lambda > 0$ is the constant multiplying the linear or angular displacement of the body.

McIver & McIver (2006) were mainly concerned with freely floating two-dimensional sections in heave motion without mooring forces so that λ arose purely from the hydrostatic restoring force. By means of an elegant application of Green's theorem they showed that, in fluid of infinite depth and with the radiation damping zero at a particular frequency, the second condition was satisfied at that frequency if and only if the potential function describing the motion had a vanishing dipole moment in the far field. This enabled them to adapt an inverse method used by McIver (1996) in constructing sloshing trapping structures to produce examples of motion trapping structures in two dimensions in the form of partially immersed heaving cylinders and their mirror images. All of their examples involved an internal free surface between the sections. They also constructed an example of a freely surging structure having three elements separated by two internal free surfaces. In a subsequent paper (McIver & McIver 2007) they extended the method to construct a three-dimensional heaving partially immersed motion trapping structure with a vertical axis of symmetry which encloses an internal free surface. Recently Evans & Porter (2007) have shown how, in two dimensions, 'mirror image' pairs of motion trapping structures can be constructed in certain parameter ranges close to frequencies at which a single element of the pair moving freely on its own is able to totally reflect an incident wave at a particular frequency. Simple geometric examples include a pair of heaving rectangular cylinders and a pair of swaying submerged tethered buoyant circular cylinders, in both cases either moving in or exactly out of phase. The authors have also produced a motion trapping structure in three dimensions in the form of a floating thick-walled cylindrical shell of rectangular cross-section. A fuller and more widely available description of these results is in preparation.

In either two or three dimensions, if there is assumed to be an external linear mooring restoring force in addition to any possible hydrostatic force, to balance the inertia forces, the construction of motion trapping structures is made easier. Thus using the same method as McIver & McIver (2006), Newman (2007) assumes the presence of a linear restoring mooring force represented by the dimensionless coefficient κ. With $\kappa = 0$ the structure is free from mooring forces and the family of motion trapping structures of McIver & McIver (2006) are recovered, whilst if $\kappa \to \pm\infty$ the structure is fixed and sloshing trapping modes are recovered. For $\kappa > 0$ the external force is opposing the motion whilst for $\kappa < 0$ the external force is driving the motion. In both cases Newman (2007) shows how new different motion trapping structures can be produced. Most of these cases involve pairs of bodies having an internal free surface between them. Significantly, however, he shows that it is possible, using a restoring force $\kappa > 0$ to construct a single body structure exhibiting motion-trapped modes. The same method could be used to construct a single axisymmetric motion trapping structure using the wave-free heaving bodies constructed by Kyozuka & Yoshida (1981) and imposing an artificial restoring force represented by κ to satisfy the second condition for trapping.

A simple geometry in two dimensions to which this method could be applied is the rolling strip described in the Introduction in either finite or infinite fluid depth. We assume that the strip is constrained to rotate about a point along its length above its mid-point and its rolling motion is resisted by an externally imposed linear couple opposing the motion. It is known that we can identify a wavenumber or frequency at which the radiation damping is zero. Then a motion trapped mode will exist if (3.1) is satisfied at that frequency. Thus in this example, ω is the frequency for which the roll radiation damping vanishes, M is the (small) moment of inertia of the strip about the point of rotation, $a(\omega)$ is the added inertia in roll due to the fluid at the zero damping frequency, and the left-hand side of the equation is positive for all frequencies. For a thin strip there is no hydrostatic force so if we choose the positive constant λ multiplying the roll amplitude to satisfy (3.1) we shall have constructed a motion trapping structure.

It is clear that all the examples described so far of single-body motion trapping structures require an artificial externally imposed restoring force to satisfy one of the conditions for trapping modes. In what follows we present the main result of the paper, which is the construction of a single submerged motion trapping structure which is free to move under natural mooring constraints.

We have shown that a submerged horizontal circular cylinder in sway or heave motions exhibits zeros of radiation damping as illustrated in figure 1 when the cylinder is sufficiently close to the free surface. But if the cylinder is buoyant and tethered to the bottom in finite depth of fluid (or to a fixed point in infinite depth of fluid) by an inelastic mooring line, then any small sway motion of the cylinder will be opposed by a natural restoring force due to the horizontal component of the tension in the cable. Evans & Linton (1989) use the same idea for a possible submerged active breakwater. More specifically, the cylinder motion is the sum of the velocity of its centre, which for small motions is a linear sway motion, plus an angular velocity about its centre which does not influence the fluid field. The required condition for a motion-trapped mode is given by (3.1) where in this particular case $\omega/2\pi$ is the wave frequency for zero radiation damping in sway, M is the mass of the cylinder, $a(\omega)$ its sway added mass and λ is the constant multiplying the sway displacement of the cylinder. It is straightforward to show that the tension in the mooring line is $T = M(1-s)g/s$ where s is the specific gravity of the cylinder so that

$$\lambda = T/l = M(1-s)g/sl, \tag{3.2}$$

where l is the length of the mooring line, from which it follows that

$$l/a = (Ka)^{-1}(1-s)/(\mu + s) \tag{3.3}$$

for a trapped mode, where $\mu = \mathrm{Re}\{a(\omega)/M\}$ is the dimensionless sway added mass of the cylinder and $Ka = \omega^2/g$. Note that μ depends on Ka and a/f where f is the depth of submergence of the centre of the cylinder, and also a/h if the fluid is of finite depth, h.

The procedure for determining a motion-trapped mode now follows from (3.3) for a cylinder in infinite depth fluid. For a fixed depth of submergence, $a/f = 0.95$, the computations leading to figures 1 and 2 provide values of Ka for which the sway damping vanishes and of the sway added mass μ at those values. With the specific gravity chosen, (3.3) now determines the required length of mooring line for a trapped mode. For example for a highly buoyant cylinder with $s = 0$, $l/a = 2.27$ for a trapped mode. Larger values of Ka for which zero damping occurs give rise to required values of $l < a$ which are less interesting. One drawback is the need to tether the cylinder in some artificial way since the fluid is infinite. However, equation (3.3) also applies in finite depth of fluid and we can seek a trapped mode when the cylinder is tethered to the bottom provided we satisfy the extra condition $l + f = h$. Thus with f and h fixed, we are no longer free to choose l to satisfy (3.3). Instead we write (3.3) in the form

$$s = (1 - \mu Kl)/(1 + Kl) \tag{3.4}$$

where the right-hand side is determined from the chosen values of a/h, f/a, and hence l/a, and the values of Ka and μ for which the sway radiation damping vanishes for the cylinder in depth h of fluid.

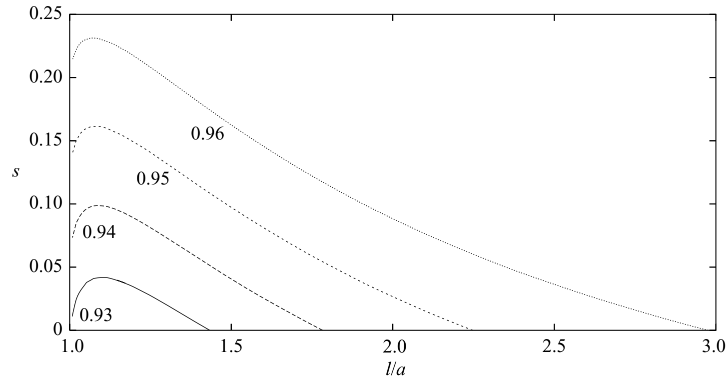

FIGURE 4. Variation of specific gravity, s, with l/a for motion-trapped modes for a cylinder, radius a, tethered with a mooring line of length l to the base of a fluid of depth h. Values lavelling curves give values of a/f, where $f = h - l$ is the submergence of the cylinder centre.

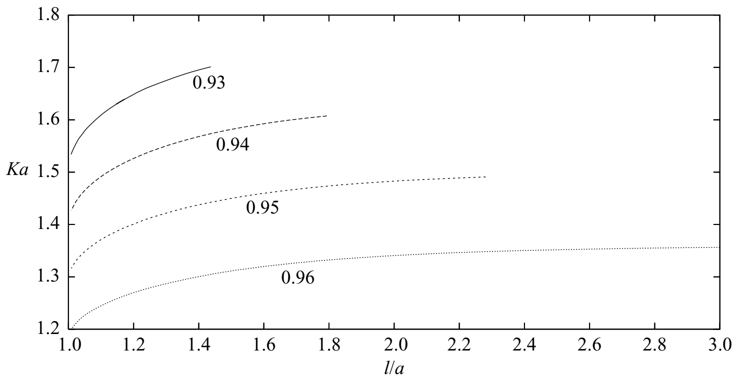

FIGURE 5. Variation of dimensionless frequency, Ka, with l/a for motion-trapped modes for a cylinder, corresponding to curves in figure 4.

Figures 4 and 5 give results for the parameters needed to generate motion-trapped modes for a cylinder tethered to the bottom of a fluid of constant finite depth h, and show the variation of both specific gravity, s, and dimensionless frequency parameter Ka with l/a, for a range of values of submergence parameter, a/f. It appears from figure 4 that we must have $a/f > 0.92$ to obtain a trapped mode with both $s \geqslant 0$ (so the cylinder is buoyant) and $l/a > 1$ (a geometrical constraint), and also that as the cylinder approaches the free surface, the range of possible values of s and l/a is widened. The ratio a/h can be determined from $a/h = 1/((a/f)^{-1} + (l/a))$.

4. Conclusion

Accurate computations of the hydrodynamic characteristics of a heaving or swaying submerged two-dimensional horizontal circular cylinder in infinite fluid depth, first computed by Ogilvie (1963), have shown that when the cylinder is sufficiently close to the free surface there exist frequencies for which both the heave and sway radiation damping coefficients, identical in an infinite fluid, vanish. This would appear to be a new result overlooked until now and further computations have shown it to be true also in finite depth where the sway and heave radiation damping coefficients are different and vanish at different frequencies. By tethering such a buoyant cylinder using an inelastic mooring line it has been shown that for frequencies at which the sway radiation damping coefficient vanishes, it is possible to choose the buoyancy and length of mooring to ensure that the second condition for motion-trapped modes to exist is satisfied. This is the first example of a submerged isolated motion trapping structure free to move under its natural mooring forces. It follows that with both conditions satisfied, when such a cylinder is displaced horizontally from its equilibrium position and then released, the eventual motion of the cylinder would, in the absence of viscous effects, be a simple harmonic motion at the trapping frequency. It would be of interest to verify this phenomenon experimentally in a narrow wave tank.

Appendix

The radiation and scattering of waves by a submerged horizontal two-dimensional circular cylinder has been considered by a number of authors. The definitive approach was due to Ursell (1950) who used the method of multipoles, which he originally developed for the solution of the heaving problem of a half-immersed circular cylinder (Ursell 1949). He considered the scattering of an incident wace by a fixed cylinder and the waves created by given pulsations of the cylinder. Ogilvie (1963) generalized this to include computations of added mass and damping coefficients whilst Linton & McIver (2001) describe the method applied to fluid of finite depth. The method in all cases reduces to solving an infinite system of algebraic equations which are rapidly convergent except when the cylinder is very close to the free surface. For details of the formulation, see the above references. Here we only present the infinite system that results from consideration of both finite and infinite fluid depth and show how their solutions are used to find the required added mass and damping coefficients.

The added mass and radiation damping coefficients, non-dimensionalized by M and $M\omega$ respectively, for a cylinder in heave (h) and sway (s) are μ^j and ν^j, $j=h,s$ respectively, and defined by

$$\mu^j + \mathrm{i}\nu^j = -1 + 2a_1^j, \qquad j = h, s, \tag{A 1}$$

in terms of complex quantities a_1^j which are the first elements of the infinite set of coefficients $\{a_n^j\}$, $n=1, 2, \ldots$ satisfying the infinite systems of equations

$$\frac{-a_m^j}{m} + \sum_{n=1}^{\infty} a_n^j A_{mn}^j = -\delta_{m1}, \tag{A 2}$$

where $\delta_{m1}=1$ if $m=1$, zero otherwise. In infinite depth, the two systems above for heave and sway are identical, since the matrix coefficients are equal:

$$A_{mn}^j = \frac{(-Ka)^{m+n}}{m!n!}\left[\int_0^{\infty} \frac{(t+1)}{(t-1)} t^{m+n-1} \mathrm{e}^{-2Kft}\,\mathrm{d}t + 2\pi\mathrm{i}\mathrm{e}^{-2Kf} \right]$$

(the integral is of Principal Value type). In terms of elementary functions, the right-hand side above equates to (see Evans *et al.* 1979)

$$\frac{(-Ka)^{m+n}}{m!n!}\left[\frac{(m+n-1)!}{(2Kf)^{m+n}} + 2\sum_{r=1}^{m+n-1} \frac{(r-1)!}{(2Kf)^r} + 2\mathrm{e}^{-2Kf}(\pi\mathrm{i} - \mathrm{Ei}(2Kf)) \right],$$

where $\mathrm{Ei}(\cdot)$ is an exponential integral.

In water of finite depth, h, the two systems in (A 2) differ, with

$$A_{mn}^h = \frac{(-Ka)^{m+n}}{n!m!}\left[\int_0^{\infty} \frac{t^{m+n-1} g_{m,n}}{(t-1) - \mathrm{e}^{-2Kht}(t+1)}\mathrm{d}t + \frac{\pi\mathrm{i}(k/K)^{m+n}}{4khN_0} f_{m,n} \right] \tag{A 3}$$

and

$$A_{mn}^s = \frac{(-Ka)^{m+n}}{n!m!}\left[\int_0^{\infty} \frac{t^{m+n-1} g_{m+1,n+1}}{(t-1) - \mathrm{e}^{-2Kht}(t+1)}\mathrm{d}t + \frac{\pi\mathrm{i}(k/K)^{m+n}}{4khN_0} f_{m+1,n+1} \right] \tag{A 4}$$

where

$$g_{m,n}(t;Kh,Kf) = (t+1)[\mathrm{e}^{-2Kft} + ((-1)^n + (-1)^m)\mathrm{e}^{-2Kht}] + (-1)^{m+n}(t-1)\mathrm{e}^{2Kft}\mathrm{e}^{-2Kht}$$

and

$$f_{m,n}(kh,kf) = \mathrm{e}^{2kh-2kf} + (-1)^n + (-1)^m + (-1)^{m+n}\mathrm{e}^{2kf-2kh}$$

whilst k is defined by $K = k\tanh kh$ and $N_0 = \frac{1}{2}(1+\sinh 2kh/2kh)$. The pole in (A 3), (A 4) is located at $t = k/K$ and the integrands are exponentially decaying at infinity. Linton & McIver (2001) show how to compute Principal Value integrals for which a standard Gaussian quadrature routine is used.

The infinite system of equations in (A 2) is approximated numerically by truncation. If $a/f \ll 1$ and Ka is not excessively large a high degree of accuracy can be achieved with very few terms (less than five). As $a/f \to 1$ and/or Ka is large an increasing number of terms are required to achieve the same accuracy.

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